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Lagrange Multiplier Modified Hořava-Lifshitz Gravity

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ABSTRACT: We consider RFDiff invariant Hořava-Lifshitz gravity action with additional Lagrange multiplier term that is a function of scalar curvature. We find its Hamiltonian formulation and we show that the constraint structure implies the same number of physical degrees of freedom as in General Relativity.

KEYWORDS: Hořava-Lifshitz gravity.

Contents

1. Introduction and Summary

In 2009 Petr Hořava formulated new proposal of quantum theory of gravity (now known as Hořava-Lifshitz gravity (HL gravity) that is power counting renormalizable [1, 2, 3] that is also expected that it reduces do General Relativity in the infrared (IR) limit [1]. The HL gravity is based on an idea that the Lorentz symmetry is restored in IR limit of given theory while it is absent in its high energy regime. For that reason Hořava considered systems whose scaling at short distances exhibits a strong anisotropy between space and time,

$$\mathbf{x}' = l\mathbf{x} , \quad t' = l^z t . \tag{1.1}$$

In $(D + 1)$ dimensional space-time in order to have power counting renormalizable theory requires that $z \geq D$. It turns out however that the symmetry group of given theory is reduced from the full diffeomorphism invariance of General Relativity to the foliation preserving diffeomorphism

$$x'^i = x^i + \zeta^i(t, \mathbf{x}) , \quad t' = t + f(t) . \tag{1.2}$$

Due to the fact that the diffeomorphism is restricted (1.2) one more degree of freedom appears that is a spin-0 graviton. It turns out that the existence of this mode could be dangerous since it has to decouple in the IR regime, in order to be consistent with observations. Unfortunately, it seems that this might not be the case. It was shown that the spin-0 mode is not stable in the original version of the HL theory [1] as well as in the Sotiriou, Visser and Weinfurtner (SVW) generalization [9]. Note that in both of these two versions, it was all assumed the projectability condition that means that the lapse function N depends on t only. This presumption has a fundamental consequence for the formulation of the theory since there is no local form of the Hamiltonian constraint but only the global one.

On the other hand we can consider the second version of HL gravity where the projectability condition is not imposed so that $N = N(\mathbf{x}, t)$ [2]. This form of HL gravity was

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[1]For review and extensive list of references, see [4, 5, 6, 7].

[2]For another proposal of renormalizable theory of gravity, see [10, 11].

extensively studied in [12, 13, 14, 15, 16, 15, 18, 19, 20, 21, 22, 23]. It was shown in [15] that so called healthy extended version of given theory could really be an interesting candidate for the quantum theory of gravity without ghosts and without strong coupling problem despite its unusual Hamiltonian structure [18, 19].

Recently Hořava and Malby-Thompson in [26] proposed very interesting way how to eliminate the spin-0 graviton in the context of the projectable version of HL gravity. Their construction is based on an extension of the foliation preserving diffeomorphism in such a way that the theory is invariant under additional local $U(1)$ symmetry. The resulting theory is known as non-relativistic covariant theory of gravity [3]. It was shown in [26, 27] that the presence of this new symmetry implies that the spin-0 graviton becomes non-propagating and the spectrum of the linear fluctuations around the background solution coincides with the fluctuation spectrum of General Relativity.

In this paper we present another version of HL gravity with the correct number of physical degrees of freedom. Our model is based on the formulation of the HL gravity with reduced symmetry group known as *restricted-foliation-preserving Diff* (RFDiff) HL gravity [15, 24]. This is the theory that is invariant under following symmetries

$$t' = t + \delta t \ , \delta t = \text{const} \ , \quad x'^i = x^i + \zeta^i(\mathbf{x}, t) \ . \tag{1.3}$$

The characteristic property of given theory is the absence of the Hamiltonian constraint [24] either global or local. Note that the meaning of the global Hamiltonian constraint is not completely clear [26] so that formulation of the HL gravity without the lapse function could be an interesting possibility how to eliminate this problem. Our construction is based on the idea of modification of RFDiff HL action that respects all symmetries of the theory however which changes the constraint structure of given theory. Remarkably this goal can be achieved when we include into the action additional term which is a function of the scalar curvature and it is multiplied by Lagrange multiplier. Then we perform the Hamiltonian analysis of given system and we show that the number of physical degrees of freedom coincides with the physical number of degrees freedom of General Relativity. This fact implies that dangerous scalar graviton is eliminated even if there is no additional gauge symmetry [4]. This remarkable result suggests that Lagrange multiplier modified RFDiff HL gravity is an interesting example of the power counting renormalizable theory of gravity with restricted symmetry group where however the scalar graviton is eliminated. On the other hand the fact that this is theory with the second class constraints makes the deeper analysis rather obscure. In fact, it is not clear how to solve the second class constraints for the physical degrees of freedom. Further, due to the fact that the Poisson bracket between second class constraints depends on the phase space variables implies that the symplectic structure on the reduced phase space defined by corresponding Dirac brackets depends on phase space variables which makes further analysis of given theory very difficult. In fact, the conventional method for covariant quantization of a theory with second class

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[3]This theory was also studied in [28, 29, 30, 31, 32, 33].

[4]We would like to stress that the Lagrange multiplier modification of RFDiff HL gravity presented in this paper can be easily extended to projectable version of HL gravity as well.

constraints is to go over to an equivalent formulation where second class constraints are replaced by the first class ones in one or another way. For example, implementing the abelian conversion of the second class constraints [46] we can formulate given theory as the theory with the first class constraints. Explicitly, by introducing additional variables Φ called conversion variables we can extend second-class constraints by Φ dependent terms such that the extended constraints become first class. However we can expect that given procedure will be purely formal due to the fact that the Poisson bracket between the second class constraints depends on the phase space variables in complicated way so that the extended Hamiltonian and constraints will contain infinite many terms.

We would like to stress that the Lagrange multiplier modified gravities were studied previously in [34], see also [35, 36]. However due to the fact that given theories are invariant under full diffeomorphism it is only possible to add to the action additional terms that are functions of the space-time curvature only. As a result the Hamiltonian structure of given theory is in agreement with basic principles of geometrodynamics [43, 44, 45]. In other words, the Hamiltonian structure of Lagrange multiplier modified $F(R)$ gravities is the same as the structure of the original $F(R)$ gravity. We can generalize this construction and consider the Lagrange multiplier modified $F(\tilde{R})$ HL gravity [37, 38], for review see [39]. Even if the resulting theory can be interesting in its own it cannot solve the scalar graviton problem of HL gravity due to the presence of additional scalar modes that are general property of all $F(R)$ theories of gravity. More precisely, the Hamiltonian structure of Lagrange multiplier modified $F(\tilde{R})$ HL gravity is the same as the Hamiltonian structure of $F(\tilde{R})$ HL gravity coupled to scalar field with specific form of the action. As a consequence the resulting constraints are not sufficient to eliminate the scalar graviton. We should however stress that we could consider yet another form of the Lagrange modified $F(\tilde{R})$ HL gravities where we add additional term that is function of the scalar curvature R instead of $\tilde{R}$. It is easy to see that the presence of the additional constraint is sufficient for the elimination of the scalar graviton.

Let us outline our results and suggest possible extension of this work. We consider Lagrange multiplier modified RFDiff invariant HL gravity and we argue that the number of physical degrees of freedom coincides with the number of degrees of freedom of General Relativity. As a consequence the scalar graviton can be eliminated in the fluctuation spectrum of given theory. If we combine this result with the well known fact that HL gravity is power counting renormalizable theory we derive an intriguing formulation of the theory of gravity that has correct number of physical degrees of freedom and which is potentially power counting renormalizable. Of course there is still the problematic fact that this is the theory with the second class constraints. The related problem is that this is the theory with the complicated symplectic structure.

Let us suggest possible extensions of given work. It would be nice to see whether the Lagrange multiplier mechanism can be implemented in the structure of infrared modified gravities (For review, see [47]) and solve some of their problems. There is also an open question how the low energy limit of the Lagrange multiplier modified RFDiff HL gravity is related to General Relativity.

This paper is organized as follows. In the next section we review basic properties

of RFDiff HL gravity and perform its modification when we include term multiplied by Lagrange multiplier into corresponding action. Then we find its Hamiltonian formulation and determine constraints structure. In section (3) we consider more general form of Lagrange multiplier modified RFDiff invariant HL gravities and analyze their properties.

2. Hořava-Lifshitz Gravity with Lagrange Multiplier

We begin this section with review of basic facts needed for the formulation of RFDiff invariant HL gravity. This is the well know $D + 1$ formalism that is the fundamental ingredient of the Hamiltonian formalism of any theory of gravity [5].

Let us consider $D + 1$ dimensional manifold $\mathcal{M}$ with the coordinates x^μ, $\mu = 0, \ldots, D$ and where $x^\mu = (t, \mathbf{x})$, $\mathbf{x} = (x^1, \ldots, x^D)$. We presume that this space-time is endowed with the metric $\hat{g}_{\mu\nu}(x^\rho)$ with signature $(-, +, \ldots, +)$. Suppose that $\mathcal{M}$ can be foliated by a family of space-like surfaces Σ_t defined by $t = x^0$. Let $g_{ij}, i, j = 1, \ldots, D$ denotes the metric on Σ_t with inverse g^{ij} so that $g_{ij}g^{jk} = \delta_i^k$. We further introduce the operator ∇_i that is covariant derivative defined with the metric g_{ij}. We introduce the future-pointing unit normal vector n^μ to the surface Σ_t. In ADM variables we have $n^0 = \sqrt{-\hat{g}^{00}}, n^i = -\hat{g}^{0i}/\sqrt{-\hat{g}^{00}}$. We also define the lapse function $N = 1/\sqrt{-\hat{g}^{00}}$ and the shift function $N^i = -\hat{g}^{0i}/\hat{g}^{00}$. In terms of these variables we write the components of the metric $\hat{g}_{\mu\nu}$ as

$$\hat{g}_{00} = -N^2 + N_i g^{ij} N_j , \quad \hat{g}_{0i} = N_i , \quad \hat{g}_{ij} = g_{ij} ,$$
$$\hat{g}^{00} = -\frac{1}{N^2} , \quad \hat{g}^{0i} = \frac{N^i}{N^2} , \quad \hat{g}^{ij} = g^{ij} - \frac{N^i N^j}{N^2} .$$

(2.1)

RFDiff invariant Hořava-Lifshitz gravity was introduced in [15] and further studied in [24]. This is the version of the Hořava-Lifshitz gravity that is not invariant under foliation preserving diffeomorphism but only under reduced set of diffeomorphism

$$t' = t + \delta t , \quad \delta t = \text{const} , \quad x'^i = x^i + \xi^i(t, \mathbf{x}) \tag{2.2}$$

The simplest form of RFDiff invariant Hořava-Lifshitz gravity takes the form [24]

$$S = \frac{1}{\kappa^2} \int dt d^D \mathbf{x} \sqrt{g} (\tilde{K}_{ij} \mathcal{G}^{ijkl} \tilde{K}_{kl} - \mathcal{V}(g)) , \tag{2.3}$$

where we introduced modified extrinsic curvature

$$\tilde{K}_{ij} = \frac{1}{2}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i) \tag{2.4}$$

that differs from the standard extrinsic curvature by absence of the lapse $N(t)$. Further the generalized De Witt metric $\mathcal{G}^{ijkl}$ is defined as

$$\mathcal{G}^{ijkl} = \frac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - \lambda g^{ij}g^{kl} , \tag{2.5}$$

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[5]For recent review, see [25].

where λ is a real constant that in case of General Relativity is equal to one. Finally $\mathcal{V}(g)$ is general function of g_{ij} and its covariant derivative.

We would like to stress that the action (2.3) differs from the projectable version of HL gravity by absence of the lapse $N = N(t)$. It is clear that we could consider more general form of RFDiff HL theory where the time and space partial derivatives of N are included [15]. Due to the fact that N behaves as scalar under (2.2) we can interpret this theory as a coupled system of the RFDiff HL gravity (2.3) with the scalar field. Then for simplicity we restrict ourselves to the action (2.3) keeping in mind that it can be easily generalized.

It was shown in [33] that this action can be extended to be invariant under $U(1)$ transformation, following very nice construction given in [26]. Hamiltonian analysis of given theory shows that the number of physical degrees of freedom coincides with the number of physical degrees of freedom of General Relativity. Now we show that the same result can be derived with the minimal extension of RFDiff HL gravity when we add to the original RFDiff HL action following term

$$S_{l.m.} = \frac{1}{\kappa^2} \int dt d^D \mathbf{x} \sqrt{g} \mathcal{G}(R) \mathcal{A} , \tag{2.6}$$

where $\mathcal{G}$ is function of $D-$dimensional curvature R [6] and where $\mathcal{A}$ is Lagrange multiplier that transforms as scalar

$$\mathcal{A}'(t', \mathbf{x}') = \mathcal{A}(t, \mathbf{x}) \tag{2.7}$$

under (2.2). Then since R and $d^D \mathbf{x} \sqrt{g}$ are manifestly invariant under (2.2) we immediately obtain that (2.6) is invariant under (2.2).

In summary we consider following action for Lagrange multiplier modified RFDiff HL gravity

$$S = \frac{1}{\kappa^2} \int dt d^D \mathbf{x} \sqrt{g} (\tilde{K}_{ij} \mathcal{G}^{ijkl} \tilde{K}_{kl} - \mathcal{V}(g) + \mathcal{G}(R) \mathcal{A}) . \tag{2.8}$$

Our goal is to perform the Hamiltonian analysis of given theory. From (2.8) we find the conjugate momenta

$$\pi^{ij} = \frac{1}{\kappa^2} \sqrt{g} \mathcal{G}^{ijkl} \tilde{K}_{kl} , \quad p^i \approx 0 , \quad p_{\mathcal{A}} \approx 0 \tag{2.9}$$

that imply the $D + 1$ primary constraints

$$p_i(\mathbf{x}) \approx 0 , \quad p_{\mathcal{A}}(\mathbf{x}) \approx 0 . \tag{2.10}$$

Further, using (2.9) we easily find the Hamiltonian with primary constraints included

$$H = \int d^D \mathbf{x} (\mathcal{H}_T + N^i \mathcal{H}_i + v^{\mathcal{A}} p_{\mathcal{A}} + v_i p^i) - \frac{1}{\kappa^2} \int d^D \mathbf{x} \sqrt{g} \mathcal{G}(R) \mathcal{A} , \tag{2.11}$$

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[6]$\mathcal{G}(R) = R - \Omega$ in [26].

where

$$\mathcal{H}_T = \frac{\kappa^2}{\sqrt{g}} \pi^{ij} \mathcal{G}_{ijkl} \pi^{kl} - \frac{1}{\kappa^2} \sqrt{g} \mathcal{V}(g) \ ,$$

$$\mathcal{H}_i = -2g_{il} \nabla_k \pi^{kl} \ .$$

$$(2.12)$$

Following standard analysis of the constraint systems [40, 41, 42] we demand that the primary constraints are preserved during the time evolution of the system. Explicitly

$$\partial_t p_\mathcal{A} = \{p_\mathcal{A}, H\} = -\frac{1}{\kappa^2} \sqrt{g} \mathcal{G}(R) \equiv -\Phi_1 \approx 0 \ ,$$

$$\partial_t p_i = \{p_i, H\} = -\mathcal{H}_i \approx 0$$

$$(2.13)$$

so that the requirement of the preservation of the primary constraints implies the secondary ones $\Phi_1 \approx 0, \mathcal{H}_i \approx 0$. Of course, now we have to demand that these constraints are preserved during the time evolution of the system. In case of $\mathcal{H}_i$ it is convenient to introduce following extended smeared form of these constraints

$$\mathbf{T}_S(N^i) = \int d^D\mathbf{x} N^i (\mathcal{H}_i + p_\mathcal{A} \partial_i \mathcal{A}) \ , \qquad (2.14)$$

where we included the primary constraint $p_\mathcal{A} \approx 0$ into the definition of $\mathbf{T}_S$. Then it is easy to see that $\mathbf{T}_S(N^i)$ is generator of spatial diffeomorphism. If we include these secondary constraints into Hamiltonian we find that the total Hamiltonian now takes the form

$$H_T = \int d^D\mathbf{x} (\mathcal{H}_T + v^\mathcal{A} p_\mathcal{A} + v_i p^i + v^1 \Phi_1) + \mathbf{T}_S(N^i) \ . \qquad (2.15)$$

Using the fact that the action is invariant under spatial diffeomorphism we immediately find that $\mathbf{T}_S(N^i)$ is preserved during the time evolution of the system. The situation is different in case of the secondary constraint Φ_1. Using following formulas

$$\{R(\mathbf{x}), \pi^{ij}(\mathbf{y})\} = -R^{ij}(\mathbf{x})\delta(\mathbf{x} - \mathbf{y}) + \nabla^i \nabla^j \delta(\mathbf{x} - \mathbf{y}) - g^{ij} \nabla_k \nabla^k \delta(\mathbf{x} - \mathbf{y}) \ ,$$

$$\nabla^i \nabla^j \mathcal{G}_{ijkl} \pi^{kl} - g^{ij} \nabla_m \nabla^m \mathcal{G}_{ijkl} \pi^{kl} = \nabla_k (\nabla_l \pi^{kl}) + \frac{1-\lambda}{\lambda D - 1} \nabla_i \nabla^i \pi$$

$$(2.16)$$

we find that the time derivative of Φ_1 is equal to

$$\partial_t \Phi_1 = \{\Phi_1, H\} \approx -2\frac{d\mathcal{G}}{dR} \left(R^{ij} \mathcal{G}_{ijkl} \pi^{kl} - \frac{1-\lambda}{\lambda D - 1} \nabla_k \nabla^k \pi \right) = 2\frac{d\mathcal{G}}{dR} \Phi_2 \ ,$$

$$(2.17)$$

where

$$\Phi_2 = -R_{ij} \pi^{ji} + \frac{\lambda}{D\lambda - 1} R\pi + \frac{1-\lambda}{\lambda D - 1} \nabla_k \nabla^k \pi \equiv M_{ij}(g(\mathbf{x})) \pi^{ji}(\mathbf{x})$$

$$(2.18)$$

is additional constraint that has to be imposed in order the constraint Φ_1 is preserved during the time evolution of the system. Following [41, 42] we include the constraint Φ_2 into definition of the total Hamiltonian so that

$$H_T = \int d^D\mathbf{x}(\mathcal{H}_T + v^{\mathcal{A}}p_{\mathcal{A}} + v_i p^i + v^1\Phi_1 + v^2\Phi_2) + \mathbf{T}_S(N^i) ,$$

(2.19)

where $v_i, v^{\mathcal{A}}, v^1, v^2$ are corresponding Lagrange multipliers.

Now we should again check the stability of all constraints. It is easy to see that the primary constraints together with $\mathbf{T}_S(N^i)$ are preserved while the time evolution of the constraint $\Phi_1 \approx 0$ is equal to

$$\partial_t\Phi_1 = \{\Phi_1, H_T\} \approx \int d^D\mathbf{x}\left(2\frac{d\mathcal{G}}{dR}\Phi_2(\mathbf{x}) + v^2(\mathbf{x})\{\Phi_1, \Phi_2(\mathbf{x})\}\right) \approx$$
$$\approx \int d^D\mathbf{x}v^2(\mathbf{x})\{\Phi_1, \Phi_2(\mathbf{x})\} = 0 .$$

(2.20)

Since

$$\{\Phi_1(\mathbf{x}), \Phi_2(\mathbf{y})\} \approx \sqrt{g}\frac{d\mathcal{G}}{dR}(R_{ij}R^{ij}(\mathbf{x})\delta(\mathbf{x} - \mathbf{y}) - \lambda R^2\delta(\mathbf{x} - \mathbf{y}) -$$
$$- \nabla^i\nabla^j\delta(\mathbf{x} - \mathbf{y})\mathcal{G}_{ijkl}R^{kl}(\mathbf{x}) - \nabla^i\delta(\mathbf{x} - \mathbf{y})\mathcal{G}_{ijkl}\nabla^j R_{kl}(\mathbf{x}) -$$
$$- \nabla^j\delta(\mathbf{x} - \mathbf{y})\mathcal{G}_{ijkl}\nabla^i R_{kl}(\mathbf{x}) - \delta(\mathbf{x} - \mathbf{y})\mathcal{G}_{ijkl}\nabla^i\nabla^j R^{kl}) +$$
$$+ \frac{1-\lambda}{D\lambda - 1}\nabla_k\nabla^k(-R(\mathbf{x})\delta(\mathbf{x} - \mathbf{y}) + (1 - D)\nabla_i\nabla^i\delta(\mathbf{x} - \mathbf{y})) \equiv$$
$$\equiv \triangle(R, R_{ij}, \mathbf{x}, \mathbf{y}) + \sqrt{g}\frac{d\mathcal{G}}{dR}\frac{(1-\lambda)(1-D)}{D\lambda - 1}\nabla_i\nabla^i\nabla_j\nabla^j\delta(\mathbf{x} - \mathbf{y})$$

(2.21)

we find that the equation (2.20) gives $v^2 = 0$. In the same way the requirement of the preservation of the constraint Φ_2 implies

$$\partial_t\Phi_2 \approx \int d^D\mathbf{x}(\{\Phi_2, \mathcal{H}_T(\mathbf{x})\} + v^1(\mathbf{x})\{\Phi_2, \Phi_1(\mathbf{x})\}) = 0 .$$

(2.22)

Using the fact that $\{\Phi_2, \mathcal{H}_T(\mathbf{x})\} \neq 0$ and also the equation (2.21) we see that (2.22) can be solved for v^1. In fact, (2.21) shows that Φ_1 and Φ_2 are the second class constraints and previous analysis that no additional constraints have to be imposed on the system. According to standard analysis the second class constraints $\Phi_A, A = 1, 2$ have to vanish strongly and allow us to express two phase space variables as functions of remaining physical phase space variable. It is important to stress that the Poisson bracket between the second class constraints depend on the phase space variables so that it is possible that it vanishes on some subspace of phase space. On the other hand from (2.21) we see that for $\lambda \neq 1$ this Poisson bracket is non-zero on the whole phase space. On the other hand in case

when $\lambda = 1$ we find that this Poisson bracket vanishes for the subspace of the phase space where $R_{ij} = 0$. First of all we have to check the consistency of the condition $R_{ij} = 0$ with the constraint $\mathcal{G}(R) = 0$. If yes, then we see that Φ_1 is preserved during the time evolution of the system and hence it is not necessary to impose additional constraint $\Phi_2 \approx 0$. Moreover, the constraint $\Phi_1 \approx 0$ is replaced with the set of more general constraints $R_{ij} \approx 0$. These constraints imply that all metric components and their conjugate momenta are non-propagating degrees of freedom and hence the theory on the subspace $R_{ij} = 0$ is effectively topological. To conclude the Poisson bracket between the constraints Φ_1, Φ_2 is non-zero on the whole phase space for $\lambda \neq 1$. In case of $\lambda = 1$ it vanishes on the subspace $R_{ij} = 0$ (on condition its consistency with constraint $\mathcal{G}(R) = 0$) which is special since it corresponds to effectively topological theory.

Returning now to the constraints Φ_1, Φ_2 we find that it is very difficult to solve them in full generality. [7] For that reason we restrict to the general discussion of the constraint structure of given theory that allows us to determine the number of physical degrees of freedom. To do this we note that there are $D(D+1)$ gravity phase space variables g_{ij}, π^{ij}, $2D$ variables N_i, p^i, 2 variables $\mathcal{A}, p_{\mathcal{A}}$. In summary the total number of degrees of freedom is $N_{D.o.f} = D^2 + 3D + 2$. On the other hand we have D first class constraints $\mathcal{H}_i \approx 0$, D first class constraints $p_i \approx 0$, one first class constraint $p_{\mathcal{A}} \approx 0$ and two second class constraints Φ_1, Φ_2. Then we have $N_{f.c.c} = 2D + 1$ first class constraints and $N_{s.c.c.} = 2$ second class constraints. As a result the number of physical degrees of freedom is [40]

$$N_{D.o.f.} - 2N_{f.c.c} - N_{s.c.c.} = D^2 - D - 2 \qquad (2.23)$$

that exactly corresponds to the number of the phase space physical degrees of freedom of $D + 1$ dimensional gravity.

We see that the phase space of the Lagrange multiplier modified RFDiff HL gravity provides correct counting of the physical degrees of freedom of gravitational theory. It is important to stress that this is the theory without global Hamiltonian constraint and without additional $U(1)$ symmetry. Further, this is the theory with non-trivial symplectic structure. To see this note that the Poisson bracket between the constraints Φ_A can be written as

$$\{\Phi_A(\mathbf{x}), \Phi_B(\mathbf{y})\} = \triangle_{AB}(\mathbf{x}, \mathbf{y}) , \qquad (2.24)$$

where the matrix $\triangle_{AB}$ has following structure

$$\triangle_{AB}(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix} , \qquad (2.25)$$

where $*$ denotes non-zero elements. It easy to see that matrix inverse to (2.25) has the form

$$(\triangle^{-1})^{AB} = \begin{pmatrix} * & * \\ * & 0 \end{pmatrix} . \qquad (2.26)$$

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[7] It is clear that the linearized approximation gives the same result as in [26] and leads to the elimination of the scalar graviton.

Now we observe that

$$\{g_{ij}(\mathbf{x}), \Phi_1(\mathbf{y})\} = 0 \ , \{g_{ij}(\mathbf{x}), \Phi_2(\mathbf{y})\} \neq 0 \ ,$$
$$\left\{\pi^{ij}(\mathbf{x}), \Phi_1(\mathbf{y})\right\} \neq 0 \ , \left\{\pi^{ij}(\mathbf{x}), \Phi_2(\mathbf{y})\right\} \neq 0 \ .$$

$$(2.27)$$

Then we find that the Dirac brackets between canonical variables take the form

$$\{g_{ij}(\mathbf{x}), g_{kl}(\mathbf{y})\}_D = -\int d\mathbf{z}d\mathbf{z}' \{g_{ij}(\mathbf{x}), \Phi_A(\mathbf{z})\} (\triangle^{-1})^{AB}(\mathbf{z}, \mathbf{z}') \{\Phi_B(\mathbf{z}'), g_{kl}(\mathbf{y})\} = 0 \ ,$$

$$\left\{\pi^{ij}(\mathbf{x}), \pi^{kl}(\mathbf{y})\right\}_D =$$
$$= -\int d\mathbf{z}d\mathbf{z}' \left\{\pi^{ij}(\mathbf{x}), \Phi_A(\mathbf{z})\right\} (\triangle^{-1})^{AB}(\mathbf{z}, \mathbf{z}') \left\{\Phi_B(\mathbf{z}'), \pi^{kl}(\mathbf{y})\right\} = \Omega^{ijkl}(\mathbf{x}, \mathbf{y}) \ ,$$

$$\left\{g_{ij}(\mathbf{x}), \pi^{kl}(\mathbf{y})\right\}_D = \left\{g_{ij}(\mathbf{x}), \pi^{kl}(\mathbf{y})\right\} -$$
$$- \int d\mathbf{z}d\mathbf{z}' \{g_{ij}(\mathbf{x}), \Phi_A(\mathbf{z})\} (\triangle^{-1})^{AB}(\mathbf{z}, \mathbf{z}') \left\{\Phi_B(\mathbf{z}'), \pi^{kl}(\mathbf{y})\right\} = \Omega_{ij}^{kl}(\mathbf{x}, \mathbf{y}) \ ,$$

$$(2.28)$$

where the matrix Ω depends on phase-space variables according to (2.26) and (2.27). The fact that the symplectic matrix depends explicitly on phase space variables implies that it is nontrivial step to proceed to the quantum mechanical analysis of given system. In principle it is possible to perform the abelian conversion of Lagrange multiplier modified RFDiff HL gravity to the system with the first class constraints following [46]. However the fact that the matrix $\triangle_{AB}$ depends on the phase space variables in non-trivial way we can expect that the resulting Hamiltonian and first class constrains will contain infinite number of terms and hence the analysis of given theory will be very complicated.

It is important to stress that the fact that the Hamiltonian is not given as linear combination of constraints has an important consequence for the stability of given theory. Explicitly, it is well known that some massive gravities are unstable since the Hamiltonian is not bounded from bellow. Alternatively, the instability of given theory is also indicated by presence of the ghosts (fields with wrong sign of kinetic term in the action) in the fluctuation spectrum. In case of the Lagrange multiplier modified RFDiff HL gravity there is no such a ghost due to the fact that only physical degrees of freedom propagate (the scalar graviton is absent) and hence linearized RFDiff invariant HL gravity has positive definite Hamiltonian. On the other hand it is not clear whether this holds in general case since in order to fully investigate the Hamiltonian of general Lagrange multiplier modified HL gravity we should solve the second class constraints and express given Hamiltonian in terms of physical modes only. However as we argued above this is very difficult task and hence an analysis of the stability of Lagrange multiplier modified HL gravity has not been performed yet.

3. More General Forms of Lagrange Multiplier Modified HL Gravities

We would like to stress that our work is based on the formulation of the Lagrange multiplier modified $F(R)$ gravities introduced in [34] where following form of Lagrange multiplier

modified $F(R)$ gravity action was considered

$$S = \int d^{D+1}x \sqrt{-\hat{g}} \left[F_1(^{(D+1)}R) - \Lambda \left(\frac{1}{2} \partial_\mu{}^{(D+1)}R \hat{g}^{\mu\nu} \partial_\nu{}^{(D+1)}R + F_2(^{(D+1)}R) \right) \right] , \quad (3.1)$$

where F_1 and F_2 are arbitrary functions and where $^{(D+1)}R$ is $D + 1$ dimensional scalar curvature and where Λ is Lagrange multiplier. Note that the action (3.1) is invariant under full diffeomorphism of the target space-time. Introducing two auxiliary fields A, B we can rewrite the action (3.1) into the form

$$S = \int d^{D+1}x \sqrt{-\hat{g}} \left[F_1(A) - \Lambda \left(\frac{1}{2} \partial_\mu A \hat{g}^{\mu\nu} \partial_\nu A + F_2(A) \right) + B(^{(D+1)}R - A) \right] \quad (3.2)$$

that is suitable for the generalization to the case of $F(R)$ HL gravity. Following [37, 38] we find the generalization of this action to the case of HL gravity when we replace $^{(D+1)}R$ with $\tilde{R}$ defined as

$$\tilde{R} = K_{ij}\mathcal{G}^{ijkl}K_{kl} + \frac{2\mu}{\sqrt{-\hat{g}}} \partial_\mu \left(\sqrt{-\hat{g}} n^\mu K \right) - \frac{2\mu}{\sqrt{g}N} \partial_i \left(\sqrt{g} g^{ij} \partial_j N \right) - \mathcal{V}(g) , \quad (3.3)$$

where μ is constant, $K = K_{ij}g^{ji}$. On the other hand using the representation (3.2) we see that the Lagrange multiplier modified $F(\tilde{R})$ HL gravity is equivalent to the $F(\tilde{R})$ HL gravity coupled to the scalar field with specific form of the action. The similar situation was analyzed in [35] in case of Lagrange multiplier modified $F(R)$ gravities. It was shown there that the Lagrange multiplier modification of the action implies specific Hamiltonian dynamics of the scalar field A while the Hamiltonian structure of the part of the action corresponding to gravity degrees of freedom is the same as in case of original $F(R)$ gravity. Clearly the same situation occurs in case of Lagrange multiplier modified $F(\tilde{R})$ HL gravity. Even if such theory could be useful for further development of the cosmological models in the context of $F(\tilde{R})$ HL gravity it is also clear that given theory cannot solve the scalar graviton problem that is general property of all projectable versions of HL gravities. However it is also clear that this scalar graviton can be eliminated when we extend $F(\tilde{R})$ HL action with the term that is function of the scalar curvature and multiplied by Lagrange multiplier. This procedure is completely the same as in previous section so that we will not repeat it here.

On the other hand we can consider more general form of Lagrange multiplier modified RFDiff HL gravity that is inspired by the action (3.1). Explicitly, let us consider following form of the Lagrange multiplier modified RFDiff HL gravity

$$S = \frac{1}{\kappa^2} \int dt d^D \mathbf{x} \sqrt{g} \left[\tilde{K}_{ij}\mathcal{G}^{ijkl}\tilde{K}_{kl} - \mathcal{V}(g) + \Lambda \left(-\frac{1}{2}\tilde{\nabla}_n R \tilde{\nabla}_n R + G \left(\frac{1}{2}g^{ij}\partial_i R \partial_j R \right) + F(R) \right) \right] , \quad (3.4)$$

where

$$\tilde{\nabla}_n = \partial_t R - N^i \partial_i R , \quad (3.5)$$

and where F and G are general functions. Introducing two auxiliary fields $A, \mathcal{A}$ we can rewrite this action in an equivalent form

$$S = \frac{1}{\kappa^2} \int dt d^D \mathbf{x} \sqrt{g} \left[\tilde{K}_{ij}\mathcal{G}^{ijkl}\tilde{K}_{kl} - \mathcal{V}(g) + \mathcal{A}(R - A) + \right.$$

$$+ \Lambda \left(-\frac{1}{2} \tilde{\nabla}_n A \tilde{\nabla}_n A + G \left(\frac{1}{2} g^{ij} \partial_i A \partial_j A \right) + F(A) \right) \bigg] \ .$$

$$(3.6)$$

The part of the action written on the first line is the same as the action (2.8) when we identify $\mathcal{G}(R)$ with $(R - A)$. In order to see whether given modification could be useful it is instructive to perform the Hamiltonian analysis of given action. As usual we begin with the definition of conjugate momenta

$$\pi^{ij} = \frac{1}{\kappa^2} \sqrt{g} \mathcal{G}^{ijkl} \tilde{K}_{kl} \ , \quad p^i \approx 0 \ , \quad p_{\mathcal{A}} \approx 0 \ ,$$
$$p_A = -\Lambda \sqrt{g} \tilde{\nabla}_n A \ , \quad p_\Lambda \approx 0 \ .$$

$$(3.7)$$

From these relations we find $D + 1$ primary constraints

$$p_i(\mathbf{x}) \approx 0 \ , \quad p_{\mathcal{A}}(\mathbf{x}) \approx 0 \ , \quad p_\Lambda(\mathbf{x}) \approx 0 \tag{3.8}$$

and the Hamiltonian

$$H = \int d^D \mathbf{x} (\mathcal{H}_T + N^i \mathcal{H}_i + v^{\mathcal{A}} \Phi_{\mathcal{A}} + v^\Lambda p_\Lambda + v_i p^i) - \frac{1}{\kappa^2} \int d^D \mathbf{x} \sqrt{g} \mathcal{A}(R - A) \ ,$$

$$(3.9)$$

where

$$\mathcal{H}_T = \mathcal{H}_T^{gr} + \mathcal{H}_T^A \ , \quad \mathcal{H}_T^{gr} = \frac{\kappa^2}{\sqrt{g}} \pi^{ij} \mathcal{G}_{ijkl} \pi^{kl} - \frac{1}{\kappa^2} \sqrt{g} \mathcal{V}(g) \ ,$$
$$\mathcal{H}_T^A = -\frac{\kappa^2}{2\Lambda \sqrt{g}} p_A^2 - \frac{1}{\kappa^2} \sqrt{g} \Lambda \left(G \left(\frac{1}{2} g^{ij} \partial_i A \partial_j A \right) + F(A) \right) \ ,$$
$$\mathcal{H}_i = -2 g_{il} \nabla_k \pi^{kl} + \partial_i A p_A \ .$$

$$(3.10)$$

Now the time evolution of the primary constraints implies

$$\partial_t p_{\mathcal{A}} = \{ p_{\mathcal{A}}, H \} = \frac{1}{\kappa^2} \sqrt{g} (R - A) \equiv \Phi_1 \approx 0 \ ,$$
$$\partial_t p_i = \{ p_i, H \} = -\mathcal{H}_i \approx 0 \ ,$$
$$\partial_t p_\Lambda = \{ p_\Lambda, H \} = -\frac{\kappa^2}{2\Lambda^2 \sqrt{g}} p_A^2 + \frac{1}{\kappa^2} \sqrt{g} \left(G \left(\frac{1}{2} g^{ij} \partial_i A \partial_j A \right) + F(A) \right) \equiv \Phi_2 \approx 0$$

$$(3.11)$$

so that we have following secondary constraints $\mathcal{H}_i \approx 0$, $\Phi_1 \approx 0$ and $\Phi_2 \approx 0$. Including these constraints into definition of the Hamiltonian we obtain the total Hamiltonian in the form

$$H = \int d^D \mathbf{x} (\mathcal{H}_T + v^i p_i + v^\Lambda p_\Lambda + v^{\mathcal{A}} p_{\mathcal{A}} + v^1 \Phi_1 + v^2 \Phi_2) + \mathbf{T}_S(N^i) \ , \tag{3.12}$$

where as usual we introduced the smeared form of the diffeomorphism constraint $\mathbf{T}_S(N^i) = \int d^D\mathbf{x}N^i(\mathcal{H}_i + p_\Lambda\partial_i\Lambda)$.

As the next step we analyze the consistency of these secondary constraints. Note that the constraint $\mathbf{T}_S(N^i)$ is preserved during the time evolution of the system from the same reason as in previous section. Further, the preservation of the constraint Φ_1 implies

$$\partial_t\Phi_1 = \{\Phi_1, H\} = -2\left(R_{ij}\pi^{ji} + \frac{\lambda}{D\lambda - 1}R\pi + \frac{(1-\lambda)}{\lambda D - 1}\nabla_k\nabla^k\pi\right) + \frac{p_A}{\Lambda} + v^2\frac{p_A}{\Lambda^2} = 0 \ .$$
(3.13)

On the other hand the preservation of the constraint $p_\Lambda \approx 0$ implies

$$\partial_t p_\Lambda = \{p_\Lambda, H\} \approx -v^2\frac{\kappa^2}{\Lambda^3\sqrt{g}}p_A^2 = 0 \ .$$
(3.14)

If we combine this equation with the equation (3.13) we find an additional constraint that has to be imposed on the system

$$\Phi_1^{II} = R_{ij}\pi^{ji} + \frac{\lambda}{D\lambda - 1}R\pi + \frac{(1-\lambda)}{\lambda D - 1}\nabla_k\nabla^k\pi + \frac{p_A}{\Lambda} \approx 0$$
(3.15)

while (3.14) determines value of the Lagrange multiplier $v_2 = 0$.

It turns out that in order to fully determine all Lagrange multipliers we have to consider the time evolution of the constraint Φ_1^{II} as well. Of course we also include the expression $v_{II}^1\Phi_1^{II}$ into the definition of the total Hamiltonian. Note also that we have following non-zero Poisson brackets

$$\left\{\Phi_1^{II}(\mathbf{x}), \mathcal{H}_T(\mathbf{y})\right\} \ , \quad \left\{\Phi_1^{II}(\mathbf{x}), \Phi_1(\mathbf{y})\right\} \ , \quad \left\{\Phi_1^{II}(\mathbf{x}), \Phi_2(\mathbf{y})\right\} \ , \quad \left\{\Phi_1^{II}(\mathbf{x}), p_\Lambda(\mathbf{y})\right\} \ , \quad (3.16)$$

where the explicit form of these Poisson brackets is not important for us. However it is clear that the presence of the additional term in the Hamiltonian has a consequence on the time evolution of all constrains. Explicitly

$$\partial_t\Phi_1 = \{\Phi_1, H\} \approx \int d^D\mathbf{x}\left(v^2(\mathbf{x})\{\Phi_1, \Phi_2(\mathbf{x})\} + v_{II}^1(\mathbf{x})\{\Phi_1, \Phi_1^{II}(\mathbf{x})\}\right) = 0 \ ,$$

$$\partial_t\Phi_2 = \{\Phi_2, H\} \approx \int d^D\mathbf{x}\left(\{\Phi_2, \mathcal{H}_T(\mathbf{x})\} + v^1(\mathbf{x})\{\Phi_2, \Phi_1(\mathbf{x})\} + \right.$$
$$\left. + v^2(\mathbf{x})\{\Phi_2, \Phi_2(\mathbf{x})\} + v_{II}^1(\mathbf{x})\{\Phi_2, \Phi_1^{II}(\mathbf{x})\} + v^\Lambda(\mathbf{x})\{\Phi_2, p_\Lambda(\mathbf{x})\}\right) \approx 0$$

$$\partial_t\Phi_1^{II} = \{\Phi_1^{II}, H\} \approx \int d^D\mathbf{x}\left(\{\Phi_1^{II}, \mathcal{H}_T(\mathbf{x})\} + v^1(\mathbf{x})\{\Phi_1^{II}, \Phi_1(\mathbf{x})\} + \right.$$
$$\left. + v^2(\mathbf{x})\{\Phi_1^{II}, \Phi_2(\mathbf{x})\} + v_{II}^1(\mathbf{x})\{\Phi_1^{II}, \Phi_1^{II}(\mathbf{x})\} + v^\Lambda\{\Phi_1^{II}, p_\Lambda(\mathbf{x})\}\right) = 0 \ ,$$

$$\partial_t p_\Lambda = \{p_\Lambda, H\} \approx \int d^D\mathbf{x}\left(v^2(\mathbf{x})\{p_\Lambda, \Phi_2(\mathbf{x})\} + v_{II}^1(\mathbf{x})\{p_\Lambda, \Phi_1^{II}(\mathbf{x})\}\right) = 0 \ .$$
(3.17)

We claim that these four equations can be solved for four unknown v^1, v^2, v_{II}^1 and v^Λ. In fact, the last equation implies the relation between v^2 and v_1^{II}

$$v_1^{II} = v^2 \frac{\kappa^2}{\Lambda \sqrt{g}} p_A \tag{3.18}$$

that together with the first equation in (3.17) implies $v^2 = v_{II}^1 = 0$. Then the second and third equations simplify considerably and can be solved for v^1, v^Λ as functions of canonical variables at least in principle. The result of this analysis is that all Lagrange multipliers are fixed. In other words we found following four second class constraints

$$\Phi_1(\mathbf{x}) \approx 0 , \quad \Phi_2(\mathbf{x}) \approx 0 , \quad \Phi_1^{II}(\mathbf{x}) \approx 0 , \quad p_\Lambda(\mathbf{x}) \approx 0 . \tag{3.19}$$

Note that these constraints can be explicitly solved on condition when we replace Poisson brackets with Dirac brackets. From the last constraint we find that $p_\Lambda(\mathbf{x}) = 0$. Further, from Φ_1 we find $A = R$ and then from Φ_2 we express Λ as

$$\Lambda^2 = \frac{\kappa^4}{2g} \frac{p_A^2}{\left(\frac{1}{2} g^{ij} \partial_i R \partial_j R + F(R)\right)} . \tag{3.20}$$

Inserting this result into $\Phi_1^{II} = 0$ we find the relation between g_{ij} and π^{ij}

$$R_{ij}\pi^{ji} + \frac{\lambda}{D\lambda - 1} R\pi + \frac{(1-\lambda)}{\lambda D - 1} \nabla_k \nabla^k \pi + \frac{\sqrt{2}}{\kappa^2} \sqrt{g} \frac{1}{\sqrt{\frac{1}{2} g^{ij} \partial_i R \partial_j R + F(R)}} = 0 . \tag{3.21}$$

Let us split the canonical momenta π^{ij} into trace and traceless parts as

$$\pi^{ij} = \tilde{\pi}^{ij} + \frac{1}{D}\pi , \quad g_{ij}\tilde{\pi}^{ji} = 0 . \tag{3.22}$$

Inserting (3.22) into (3.21) we can presume that it can be solved for π at least in principle. Then the reduced phase space is spanned by $g_{ij}, \tilde{\pi}^{ij}$ and p_A so that we have $D(D+1)$ physical degrees of freedom.

Generally we can determine the number of physical degrees as follows. We have $D(D+1)$ metric phase space variables g_{ij}, π^{ij}, $2D$ phase space variables N_i, p^i, 6 phase space variables $\Lambda, p_\Lambda, A, p_A, \mathcal{A}, p_\mathcal{A}$. In summary we have $N_{D.o.f.} = D^2 + 3D + 6$ phase space degrees of freedom. On the other hand we have $2D$ first class constraints $\mathcal{H}_i \approx 0, p^i \approx 0$, one first class constraint $p_\mathcal{A} \approx 0$ and 4 second class constraints $p_\Lambda \approx 0, \Phi_1 \approx 0, \Phi_2 \approx 0$ and $\Phi_1^{II} \approx 0$. In summary we have $N_{f.c.c.} = 2D + 1$ first class constraints and $N_{s.c.c.} = 4$ second class constraints. Then the number of physical degrees of freedom is

$$N_{D.o.f.} - 2N_{f.c.c.} - N_{s.c.c.} = D^2 - D = (D^2 - D - 2) + 2 \tag{3.23}$$

where the expression in parenthesis determines the number of physical degrees of freedom of massless graviton while the remaining part corresponds to the scalar mode that is present in the theory. In other words the generalized Lagrange multiplier modified RFDiff HL gravity is not sufficient for the elimination of the scalar graviton and should be only considered as an interesting example of the theory with reduced symmetry group.

References

[1] P. Horava, *"Quantum Gravity at a Lifshitz Point,"* Phys. Rev. D **79** (2009) 084008 [arXiv:0901.3775 [hep-th]].

[2] P. Horava, *"Membranes at Quantum Criticality,"* JHEP **0903** (2009) 020 [arXiv:0812.4287 [hep-th]].

[3] P. Horava, *"Quantum Criticality and Yang-Mills Gauge Theory,"* arXiv:0811.2217 [hep-th].

[4] P. Horava, *"General Covariance in Gravity at a Lifshitz Point,"* arXiv:1101.1081 [hep-th].

[5] A. Padilla, *"The good, the bad and the ugly .... of Horava gravity,"* arXiv:1009.4074 [hep-th].

[6] S. Mukohyama, *"Horava-Lifshitz Cosmology: A Review,"* arXiv:1007.5199 [hep-th].

[7] S. Weinfurtner, T. P. Sotiriou and M. Visser, *"Projectable Horava-Lifshitz gravity in a nutshell,"* J. Phys. Conf. Ser. **222**, 012054 (2010) [arXiv:1002.0308 [gr-qc]].

[8] T. P. Sotiriou, *"Horava-Lifshitz gravity: a status report,"* arXiv:1010.3218 [hep-th].

[9] T. P. Sotiriou, M. Visser and S. Weinfurtner, *"Quantum gravity without Lorentz invariance,"* JHEP **0910** (2009) 033 [arXiv:0905.2798 [hep-th]].

[10] S. Nojiri and S. D. Odintsov, *"Covariant Horava-like renormalizable gravity and its FRW cosmology,"* Phys. Rev. D **81** (2010) 043001 [arXiv:0905.4213 [hep-th]].

[11] S. Nojiri and S. D. Odintsov, *"A proposal for covariant renormalizable field theory of gravity,"* Phys. Lett. B **691** (2010) 60 [arXiv:1004.3613 [hep-th]].

[12] D. Blas, O. Pujolas and S. Sibiryakov, *"On the Extra Mode and Inconsistency of Horava Gravity,"* JHEP **0910** (2009) 029 [arXiv:0906.3046 [hep-th]].

[13] D. Blas, O. Pujolas and S. Sibiryakov, *"A healthy extension of Horava gravity,"* arXiv:0909.3525 [hep-th].

[14] D. Blas, O. Pujolas and S. Sibiryakov, *"Comment on 'Strong coupling in extended Horava-Lifshitz gravity',"* arXiv:0912.0550 [hep-th].

[15] D. Blas, O. Pujolas and S. Sibiryakov, *"Models of non-relativistic quantum gravity: the good, the bad and the healthy,"* arXiv:1007.3503 [hep-th].

[16] M. Li and Y. Pang, *"A Trouble with Hořava-Lifshitz Gravity,"* JHEP **0908** (2009) 015 [arXiv:0905.2751 [hep-th]].

[17] M. Henneaux, A. Kleinschmidt and G. L. Gomez, *"A dynamical inconsistency of Horava gravity,"* arXiv:0912.0399 [hep-th].

[18] J. Kluson, *"Note About Hamiltonian Formalism of Modified $F(R)$ Hořava-Lifshitz Gravities and Their Healthy Extension,"* Phys. Rev. D **82** (2010) 044004 [arXiv:1002.4859 [hep-th]].

[19] J. Kluson, *"Note About Hamiltonian Formalism of Healthy Extended Horava-Lifshitz Gravity,"* JHEP **1007** (2010) 038 [arXiv:1004.3428 [hep-th]].

[20] J. Bellorin and A. Restuccia, *"Closure of the algebra of constraints for a non-projectable Hořava model,"* arXiv:1010.5531 [hep-th].

[21] J. Bellorin and A. Restuccia, *"On the consistency of the Horava Theory,"* arXiv:1004.0055 [hep-th].

[22] A. Kobakhidze, *"On the infrared limit of Horava's gravity with the global Hamiltonian constraint,"* Phys. Rev. D **82** (2010) 064011 [arXiv:0906.5401 [hep-th]].

[23] J. M. Pons and P. Talavera, *"Remarks on the consistency of minimal deviations from General Relativity,"* Phys. Rev. D **82** (2010) 044011 [arXiv:1003.3811 [gr-qc]].

[24] J. Kluson, *"Horava-Lifshitz Gravity And Ghost Condensation,"* Phys. Rev. D **82** (2010) 124011 [arXiv:1008.5297 [hep-th]].

[25] E. Gourgoulhon, *"3+1 Formalism and Bases of Numerical Relativity,"* arXiv:gr-qc/0703035.

[26] P. Horava and C. M. Melby-Thompson, *"General Covariance in Quantum Gravity at a Lifshitz Point,"* Phys. Rev. D **82** (2010) 064027 [arXiv:1007.2410 [hep-th]].

[27] A. M. da Silva, *"An Alternative Approach for General Covariant Horava-Lifshitz Gravity and Matter Coupling,"* arXiv:1009.4885 [hep-th].

[28] J. Greenwald, V. H. Satheeshkumar and A. Wang, *"Black holes, compact objects and solar system tests in non-relativistic general covariant theory of gravity,"* arXiv:1010.3794 [hep-th].

[29] J. Alexandre and P. Pasipoularides, *"Spherically symmetric solutions in Covariant Horava-Lifshitz Gravity,"* arXiv:1010.3634 [hep-th].

[30] A. Wang and Y. Wu, *"Cosmology in nonrelativistic general covariant theory of gravity,"* arXiv:1009.2089 [hep-th].

[31] Y. Huang and A. Wang, *"Nonrelativistic general covariant theory of gravity with a running constant λ,"* arXiv:1011.0739 [hep-th].

[32] J. Kluson, S. Nojiri, S. D. Odintsov and D. Saez-Gomez, *"U(1) Invariant F(R) Horava-Lifshitz Gravity,"* arXiv:1012.0473 [hep-th].

[33] J. Kluson, *"Hamiltonian Analysis of Non-Relativistic Covariant RFDiff Horava-Lifshitz Gravity,"* arXiv:1011.1857 [hep-th].

[34] S. Capozziello, J. Matsumoto, S. 'i. Nojiri *et al.*, *"Dark energy from modified gravity with Lagrange multipliers,"* Phys. Lett. **B693** (2010) 198-208. [arXiv:1004.3691 [hep-th]].

[35] J. Kluson, *"Hamiltonian Analysis of Lagrange Multiplier Modified Gravity,"* [arXiv:1009.6067 [hep-th]].

[36] Y. -F. Cai, E. N. Saridakis, *"Cyclic cosmology from Lagrange-multiplier modified gravity,"* [arXiv:1007.3204 [astro-ph.CO]].

[37] S. Carloni, M. Chaichian, S. 'i. Nojiri *et al.*, *"Modified first-order Horava-Lifshitz gravity: Hamiltonian analysis of the general theory and accelerating FRW cosmology in power-law F(R) model,"* Phys. Rev. **D82** (2010) 065020. [arXiv:1003.3925 [hep-th]].

[38] M. Chaichian, S. 'i. Nojiri, S. D. Odintsov *et al.*, *"Modified F(R) Horava-Lifshitz gravity: a way to accelerating FRW cosmology,"* Class. Quant. Grav. **27** (2010) 185021. [arXiv:1001.4102 [hep-th]].

[39] S. 'i. Nojiri, S. D. Odintsov, *"Unified cosmic history in modified gravity: from F(R) theory to Lorentz non-invariant models,"* [arXiv:1011.0544 [gr-qc]].

[40] M. Henneaux and C. Teitelboim, *"Quantization of gauge systems,"* Princeton, USA: Univ. Pr. (1992) 520 p

[41] J. Govaerts, *"The quantum geometer's universe: Particles, interactions and topology,"* arXiv:hep-th/0207276.

[42] J. Govaerts, *"Hamiltonian Quantization And Constrained Dynamics,"* Leuven, Belgium: Univ. Pr. (1991) 371 p. (Leuven notes in mathematical and theoretical physics, B4)

[43] C. J. Isham and K. V. Kuchar, *"Representations Of Space-Time Diffeomorphisms. 1. Canonical Parametrized Field Theories,"* Annals Phys. **164** (1985) 288.

[44] C. J. Isham and K. V. Kuchar, *"Representations Of Space-Time Diffeomorphisms. 2. Canonical Geometrodynamics,"* Annals Phys. **164** (1985) 316.

[45] S. A. Hojman, K. Kuchar and C. Teitelboim, *"Geometrodynamics Regained,"* Annals Phys. **96** (1976) 88.

[46] I. A. Batalin and I. V. Tyutin, *"Existence theorem for the effective gauge algebra in the generalized canonical formalism with Abelian conversion of second class constraints,"* Int. J. Mod. Phys. A **6** (1991) 3255.

[47] V. A. Rubakov and P. G. Tinyakov, *"Infrared-modified gravities and massive gravitons,"* Phys. Usp. **51** (2008) 759 [arXiv:0802.4379 [hep-th]].